Solution of Electromagnetic Scattering Problems Using an Eigenmode Projection Technique

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***Abstract*—Dielectric objects illuminated by uniform plane waves are analyzed using an eigenmode projection technique. A fictitious canonical cavity is chosen to enclose the scatterer, and the fields are expanded in terms of the cavity solenoidal and irrotational eigenmodes. The fictitious cavity surface is regarded as a port excited by the uniform plane wave, and the total cavity fields and the port fields are then matched on the surface resulting in the field coefficients. Results obtained using the proposed method are compared with results obtained for problems of known analytical solution and results obtained using other numerical techniques.**

I. Introduction

Numerical techniques used in solving electromagnetic problems are mainly based on geometrical discretization and the choice of some basis functions to solve the problem under consideration. However, the chosen basis functions do not have any physical indication. In [1], an approach based on eigenmode projection technique was adopted in the analysis of microwave cavities. In this work, the eigenmode projection technique is employed to solve problems of electromagnetic scattering in free space. In the proposed method, a fictitious canonical cavity is chosen to enclose the scatterer, and the fields are expanded in terms of the cavity solenoidal and irrotational eigenmodes. The fictitious cavity surface is regarded as a port excited by the uniform plane wave, and the total cavity fields and the port fields are then matched on the surface resulting in the field coefficients. In the next sections, details of the proposed technique are outlined and some results are presented to validate the method.

II. Problem Formulation

An arbitrary dielectric object is excited by uniform plane wave as shown in Fig. 1. A fictitious canonical cavity with perfect magnetic (PM) or perfect electric (PE) boundary is chosen to provide the set of eigenmodes that is used in solving the problem. The cavity is chosen to be either cylindrical or spherical according to the problem under consideration either being two dimensional (2D) or three dimensional (3D), respectively.

The total port field is the summation of the incident field and the scattered port modes as:

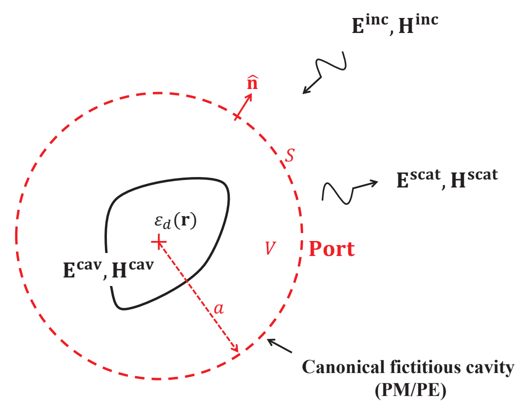

Fig. 1. General scattering problem by an arbitrary dielectric object with permittivity function $\varepsilon_d(\mathbf{r})$, the object is enclosed by a canonical fictitious cavity with radius a , volume V and outer surface S.

$$\mathbf{E^{port}}(\mathbf{r}) = \mathbf{E^{inc}}(\mathbf{r}) + \mathbf{E^{scat}}(\mathbf{r}) \tag{1}$$

$$\mathbf{H^{port}}(\mathbf{r}) = \mathbf{H^{inc}}(\mathbf{r}) - \frac{1}{j\omega\mu_0}\nabla \times \mathbf{E^{scat}}(\mathbf{r}) \tag{2}$$

where $\mathbf{E^{scat}}(\mathbf{r}) = \sum_p a_p^{scat}\mathbf{D}_p(\mathbf{r})$, with $\mathbf{D}_p(\mathbf{r})$ being the functional dependence of the scattered fields from cylindrical or spherical objects in case of 2D or 3D problems, respectively [2].

Cavity eigenmodes are a set of orthogonal solenoidal ($\mathbf{E}_n$, $\mathbf{H}_n$), divergence free, and irrotational ($\mathbf{F}_\alpha$, $\mathbf{G}_\lambda$), curl free, modes. The solenoidal modes are related together by $\nabla \times \mathbf{E}_n = k_n\mathbf{H}_n$ and $\nabla \times \mathbf{H}_n = k_n\mathbf{E}_n$, however the irrotational modes are represented by the divergence of scalar potentials $l_\alpha\mathbf{F}_\alpha = \nabla\phi_\alpha$ and $l_\lambda\mathbf{G}_\lambda = \nabla\psi_\lambda$, where k_n, l_α, l_λ are wave numbers for solenoidal, irrotational electric and magnetic fields respectively [3, 4]. Thus the cavity fields could be expressed as follows:

$$\mathbf{E^{cav}}(\mathbf{r}) = \sum_n a_n\mathbf{E}_n(\mathbf{r}) + \sum_\alpha f_\alpha\mathbf{F}_\alpha(\mathbf{r}) \tag{3}$$

$$\mathbf{H^{cav}}(\mathbf{r}) = \sum_n b_n\mathbf{H}_n(\mathbf{r}) + \sum_\lambda g_\lambda\mathbf{G}_\lambda(\mathbf{r}) \tag{4}$$

The canonical cavity is chosen with PM boundary to eliminate the irrotational magnetic field [1]. Applying fields expansions to Maxwell's equations and performing modes projection leads after some lengthy algebraic manipulations to the following equations:

$$k_n a_n = -j\omega\mu_0 b_n \tag{5}$$

$$k_n b_n + \oint_S \left(\mathbf{H^{port}}(a) \times \mathbf{E}_n(a)\right) \cdot ds = j\omega \left[\sum_m \langle \mathbf{E}_n, \mathbf{E}_m \rangle a_m + \sum_{\alpha'} \langle \mathbf{E}_n, \mathbf{F}_{\alpha'} \rangle f_{\alpha'}\right] \tag{6}$$

$$\oint_S \left(\varepsilon(a)\mathbf{E^{port}}(a)\phi_\alpha(a)\right) \cdot ds - l_\alpha \left[\sum_m \langle \mathbf{F}_\alpha, \mathbf{E}_m \rangle a_m + \sum_{\alpha'} \langle \mathbf{F}_\alpha, \mathbf{F}_{\alpha'} \rangle f_{\alpha'}\right] = 0 \tag{7}$$

where, $\langle \mathbf{X}, \mathbf{Y} \rangle = \int_V \varepsilon(\mathbf{r})\mathbf{X}(\mathbf{r}) \cdot \mathbf{Y}(\mathbf{r}) dv$.

Substituting from (1) and (2) in (7) and (6), respectively then combining the equations (5-7) yields a matrix equation of the form:

$$[A_{nm}]\,[a_n] + [B_{np}]\,\left[a_p^{scat}\right] = [C_n] \tag{8}$$

where, n, and $m = 1, 2, \cdots N$ and $p = 1, 2, \cdots M$ with N and M being the number of cavity and port modes under consideration, respectively.

In order to evaluate the scattered fields coefficients another equation is obtained by enforcing the boundary conditions by equating the tangential electric fields components for port and cavity modes at cavity boundary, this is done in average sense by performing surface projecting over $\mathbf{D}_{p'}$ to avoid discontinuity at the surface as follows:

$$\oint_S \hat{\mathbf{n}} \times \mathbf{E^{port}}(a).\mathbf{D}_{p'}(a)ds = \oint_S \hat{\mathbf{n}} \times (\sum_n a_n \mathbf{E}_n(a) + \sum_\alpha f_\alpha \mathbf{F}_\alpha(a)).\mathbf{D}_{p'}(a)ds \tag{9}$$

Combining the equations (1), (7) and (9) another equation similar to (8) is obtained and consequently the scattered field coefficients $\left[a_p^{scat}\right]$.

III. Results and Verification

Bistatic Scattering width (SW) [2] of 2D dielectric objects illuminated by transverse magnetic (TM^z) plane wave is used to verify the proposed technique.

In Fig. 2 results obtained for cylindrical scatterer are compared with the analytical solution in [2], showing that increasing number of modes makes the solution more close to analytical curve.

In Fig. 3 results obtained for rectangular scatterer are compared with the results obtained using the method of moments (MOM).

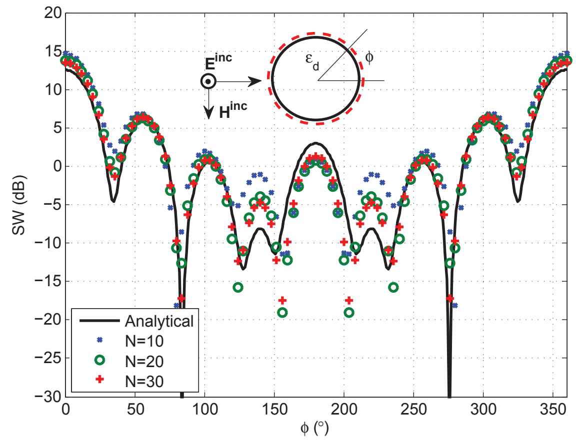

Fig. 2. SW for cylindrical dielectric scatterer with $\varepsilon_d = 3\varepsilon_0$ and radius $= 0.6\lambda$ illuminated by TM^z plane wave with canonical cavity touching the dielectric surface compared with the analytical solution.

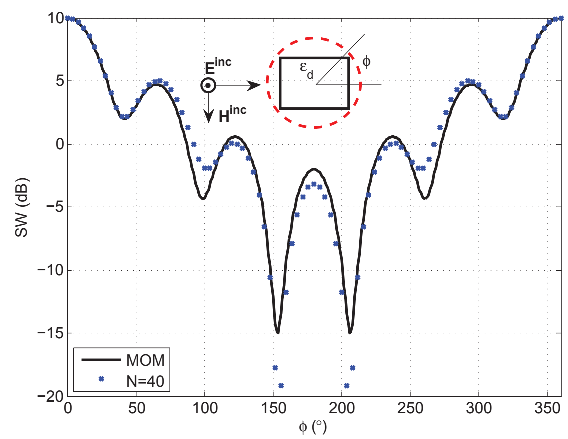

Fig. 3. SW for rectangular dielectric scatterer with $\varepsilon_d = 4\varepsilon_0$ and dimensions $0.8\lambda \times 0.6\lambda$ illuminated by TM^z plane wave incident perpendicular to its short side with canonical cavity passing through the rectangle corners compared with the results obtained using MOM.

IV. Conclusion

Eigenmode projection technique has the advantage over other numerical techniques that it uses physically inspired modes to solve electromagnetic problems. Moreover it has clear methodology in increasing the efficiency by increasing the number of modes used in the solution. Adopting this technique in the dielectric scattering problems helps in a variety of applications.

References

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