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Circuit-Energy Aware Discrete Bit Loading

Omar A. Nasr, and Babak Daneshrad
Wireless Integrated Research Lab, Electrical Engineering department, UCLA

Abstract— **Two fundamental problems related to multi-carrier modulation are considered in this paper. The first is finding the best modulation schemes in all subcarriers to minimize the total energy consumption required to send a given number of bits between two wireless nodes. The second is finding the best modulation schemes to maximize the number of bits transmitted between two nodes under total energy constraint. Discrete bit loading algorithms have been used to solve both problems over frequency selective channels. None of these algorithms takes the circuit energy into consideration. In this paper, we introduce new discrete bit loading algorithms to solve both problems with total energy defined as the sum of transmission energy and circuit energy. Compared to the conventional discrete bit loading algorithms, the new algorithms can achieve savings in the order of 9 dB in the energy minimization problem. 180% increase in the number of transmitted bits can also be achieved with the same total energy when compared to the existing algorithms.**

I. Introduction

A fundamental problem in communications is finding the optimal distribution of a limited total energy among a set of parallel AWGN channels with different channel gains in order to maximize the total number of bits delivered from one node to another. The solution is well known in the literature by "water-filling" [1]. When the bits assignment is constrained to be integers, i.e. in QAM systems, the water-filling algorithm is modified to meet this requirement. Depending on the system under consideration, the margin maximization problem or the rate maximization problem is considered [2]. In the margin maximization problem, the goal of the transmitter is to deliver a given number of bits by using the minimum possible energy under a time constraint. The dual problem is the rate maximization problem, where the transmitter tries to maximize the total number of transmitted bits using a limited energy budget under a time constraint. The algorithms used to solve these problems are called "discrete bit loading" algorithms. In [3], necessary and sufficient conditions for the optimal solutions of both problems have been proposed. In [2], greedy algorithms have been proposed based on these conditions and additional constraints on the maximum number of bits for each subcarrier. This area of research has flourished due to the popularity of its application in the area of Discrete Multi-Tone (DMT) modulation for the ADSL systems [4]. For frequency selective wireless links with slow channel variations, these algorithms can also be implemented when the channel state information is available on the transmitter side.

On a parallel track, another problem that researchers have considered is the total energy constrained modulation optimization [5]. In this problem, the total energy, including both transmission energy and circuit energy, is considered in the energy minimization problem. In [5], they showed that the transmission energy is the bottleneck only at large distances, but for smaller distances, the circuit energy dominates the total energy required for transmission. Several new wireless technologies use distance transmission, such as sensor networks and personal area networks. Although further research has been pursued based on the total energy consumption, e.g. [6], the effect of circuit energy on the bit loading algorithms in frequency selective channels has not been studied. In [7], a new objective function was introduced to take the effect of circuit energy into consideration, but the authors have not addressed the major two problems of interest, namely the margin maximization and the rate maximization problems.

In this paper, we consider the two problems of margin and rate maximization in a multi carrier system over a frequency selective channel. Unlike all previous discrete bit loading research papers, the total energy in this paper takes into account both transmission energy and circuit energy. Direct application of discrete bit loading algorithms in the literature can not solve the problem when circuit energy is considered. Therefore, we introduce novel bit loading algorithms suitable to solve both problems. For the margin maximization problem, we show that energy saving in order of 9 dB is possible using our algorithm compared to the conventional discrete bit loading algorithms. Large gains in order of 180% are also possible in the case of rate maximization problem by using our algorithm.

The remainder of this paper is organized as follows: Section II describes the system model. Section III formalizes the circuit energy aware margin maximization problem and introduces a greedy algorithm to solve the problem. In Section IV, we define the circuit energy aware rate maximization problem and its solution is explained. In Section V, we show the potential gain of our algorithm in numerical examples for both problems. Our conclusions are summarized in section IV.

II. System Model

We consider the communication link between one transmitter and one receiver. The data is transmitted using a multicarrier modulation over M orthogonal subcarriers. Throughout the paper, we will assume OFDM transmission,

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although DMT can also be used. The channel between the two nodes is assumed to be a frequency selective channel. We also assume a full knowledge of channel state information at the transmit side. The received signal at subcarrier k and time i can be written as:

$$Y_k[i] = \sqrt{e_{ik}(b_{ik})}X_k[i]\frac{H_k[i]}{\sqrt{G_d[i]}} + N[i] \quad (1)$$

where $N[i]$ is the noise at any subcarrier. The noise over all subcarriers is assumed to have the same power spectral density σ_n^2. For ease of notation, we will drop the time index i unless it is needed to be mentioned explicitly. The instantaneous channel between the transmitter and the receiver at subcarrier k is $H_k/\sqrt{G_d}$, where G_d is the power gain factor and H_k is the frequency selective channel component. At distance d between the transmitter and the receiver, the transmitted power will be attenuated by a factor of G_d at any subcarrier due to the path loss attenuation. We will use the same model as [5], where $G_d = G_1 d^{\kappa} M_l$ with M_l is the link margin, G_1 is the gain factor at d=1m and κ is the environment path loss exponent. Due to the selectivity of the channel, there will be a frequency selective component (H_k). For ease of illustration, we assume a channel with an exponentially increasing gain in the frequency domain. The selectivity (S) of the channel is defined as the relative gain between the first subcarrier and subcarrier M,

$$S(dB) = 10\log_{10}(|H_k/H_1|^2), \quad (2)$$

which is illustrated in Fig 1. For fair comparison between different channels with different selectivities, the frequency selective channel component will be normalized over all subcarriers:

$$\sum_{k=1}^{M}|H_k|^2 = 1. \quad (3)$$

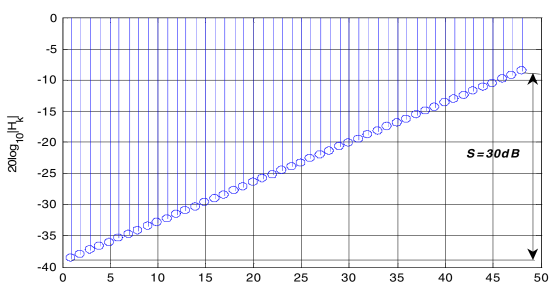

Fig. 1. A frequency selective channel with selectivity = 30 dB.

The transmitter sends uncoded QAM symbols (X_k) with different number of bits at different subcarriers. The number of bits per subcarrier is b_k, where $0<b_k<b^{max}$ and b^{max} is the maximum number of bits that can be transmitted in any subcarrier. The transmitter may choose not to send any bits over subcarrier k by setting b_k=0. The QAM symbols are normalized to have a unity average power $E(|X_k|^2)=1$.

The QAM symbols are transmitted with different energies $e_k(b_k)$ over different subcarriers. A target Bit Error Rate (BER) is required to be achieved at all subcarriers. The transmission energy per subcarrier required to achieve a certain BER is:

$$e_k(b_k) = \frac{G_d}{|H_k|^2}E_{b_k}(BER), \quad (4)$$

where $E_{b_k}(BER)$ is energy required to be received at subcarrier k to deliver b_k bits with the target BER. In our system, we assume that the transmitter can transmit the data over L OFDM symbols, which is called a "signaling period". As a result, the total number of subcarriers is ($M.L$). The channel is assumed to be constant during the entire signaling period. The total energy required to send a packet over L OFDM symbols is:

$$E_{tot} = \sum_{i=1}^{L}\sum_{k=1}^{M} e_{ik}(b_{ik}) + L.E_c = E_t + L.E_c, \quad (5)$$

where E_c is the circuit energy required to send one OFDM symbol and E_t is the transmission energy. In our model, we will assume that the circuit power during the idle period is negligible. The circuit energy consists of both baseband energy and RF energy. A detailed system level energy model for all the components in the RF front-end is presented in [9]. The baseband energy depends on the system under consideration, the technology used, and the implementation architecture. For example, [10] reports 795 mW for the transmit side of an 802.11a system. Fig 2 shows the general trade-off between the two energy components. The circuit energy component increases linearly with the number of OFDM symbols used for transmission. On the other hand, the transmission energy decreases with using more OFDM symbols. The main reason is that transmitting over more OFDM symbols will increase the number of populated subcarriers ($M.L$), which will decrease the average number of bits per subcarrier and consequently will decrease the transmitted energy to reach the target BER per subcarrier. For the total energy, there will be always an optimal number of symbols to use l^* and a corresponding bit distribution over the subcarriers in the used symbols that minimizes the overall energy required to transmit the packet.

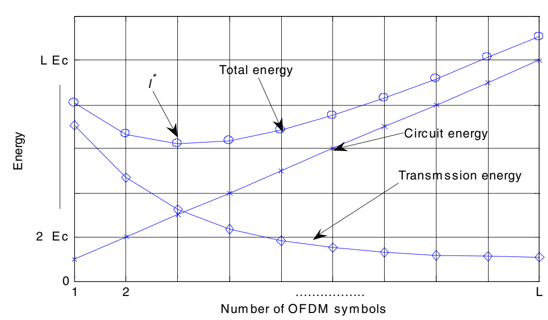

Fig. 2. General trend of transmission and circuit energies.

III. MARGIN MAXIMIZATION PROBLEM

The standard margin maximization discrete bit allocation problem is stated as follows [2]:

$$\min_{0\le b_k \le b^{\max}} \sum_{k=1}^{k=M} e_k(b_k)$$
$$s.t \qquad (6)$$
$$\sum_{k=1}^{k=M} b_k = R; \quad b_k \le b^{\max}, \quad k=1,....,M$$

where R is total number of bits required to be sent in an OFDM symbol. The basic solution of the margin maximization problem is explained in details in [2]. Other algorithms have been developed, but they are all modifications to the greedy algorithm in [2]. The steps to solve the margin maximization problem are explained below:

Algorithm 1:

1. $b_k = 0, \; k = 1,...,M$
2. while $\sum_{k=1}^{M} b_k < R$
3. find $k^* \in \{1,...,M\}$ s.t
4. $(1) b_{k^*} < b^{\max}$
5. $(2) \Delta e_{k^*}(b_{k^*}+1) = \min_{k:b_k<b^{\max}} \Delta e_k(b_k+1)$
6. set $b_{k^*} = b_{k^*}+1$
7. end.

where $\Delta e_k(b_k)$ is the energy required to send an extra bit at subcarrier k:

$$\Delta e_k(b_k) = e_k(b_k) + e_k(b_k - 1), \; b_k \ge 1 \qquad (7)$$
$$e_k(0) = 0 ,$$

Having the data transmitted over multiple OFDM symbols within a signaling period will not make any change in the solution if the circuit energy is not considered. The greedy algorithm will always fill the high gain channels in all OFDM symbols before using the worse channels. Fig. 3 shows the bit distribution per OFDM symbol in case of using one symbol versus using 10 symbols. In this example, R=196 bits, $b^{\max}$=8 and S=20 dB. We also show the transmission energy required to transmit all bits using different number of OFDM symbols. The energies are shown relative to the minimum possible energy when using 10 symbols. A savings of about 12 dB can be achieved when using all 10 symbols compared to using only one symbol. Note that the bit distribution when using 10 symbols will *almost* be identical for all symbols. Because we limit the number of bits per subcarrier to be integer, we can not send 19.6 bits per OFDM symbol. As a result, 6 out of the 10 symbols will transmit 20 bits and the rest will transmit 19 bits.

As mentioned before, using more symbols will lead to a higher total circuit energy. As a result, the circuit energy-aware problem is defined as:

$$\min_{b_{ik},l} E_{tot} = E_t + lE_c$$
$$s.t \;\; (1)\; E_t = \sum_{i=1}^{L}\sum_{k=1}^{M} e_{ik}(b_{ik}) \qquad (8)$$
$$(2) \sum_{i=1}^{L}\sum_{k=1}^{M} b_{ik} = R; \; b_{ik} \le b^{\max}, \;\; k=1,..,M, \; i=1,..,L$$

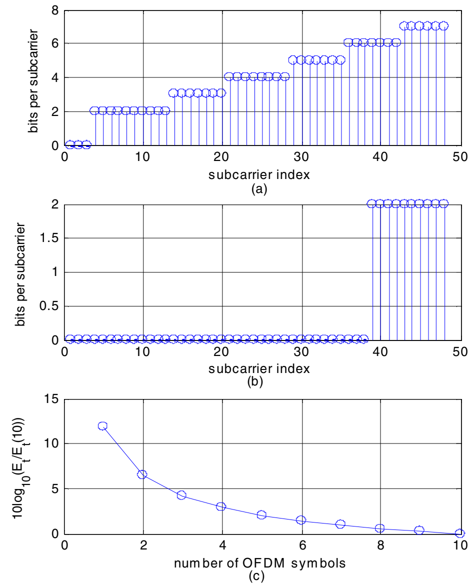

Fig. 3. (a) Bit distribution *per symbol* when using one symbol and (b) 10 symbols. (c) Transmission energy for number of OFDM symbols.

The new margin maximization problem has one more parameter to optimize than the conventional problem, which is the optimal number of OFDM symbols to be used to transmit the packet. Algorithm 1 can not determine the best number of OFDM symbols to use. A solution for the optimization problem in (8) can be obtained based on Lemma 1.

Lemma 1: If l OFDM symbols *must* be used to transmit the packet, then the distribution of bits over the subcarriers on all OFDM symbols can be obtained using Algorithm 1.

Proof: If l OFDM symbols are used, then the number of symbols is *not* one of the parameters to optimize. Therefore, the optimization problem becomes:

$$\min E_{tot} = E_t + lE_c \longrightarrow \min E_t \longrightarrow \text{discrete bit loading problem}$$
$$\min \sum_{i=1}^{l}\sum_{k=1}^{M} e_{ik}(b_{ik})$$
$$s.t \;\; \sum_{i=1}^{l}\sum_{k=1}^{M} b_{ik} = R; \; b_{ik} \le b^{\max}, \;\; k=1,..,M, \; i=1,..,l$$

Based on the previous Lemma, a circuit energy aware bit loading algorithm that can solve the margin maximization problem is explained below:

Algorithm 2:

1. $b_k = 0; 1 \le k \le M$
2. $l = L$
3. $E_l = 0; 1 \le l \le L$
4. $E_{acc} = 0$
5. while $(l > 0)$
6. $R_l = \lceil R/l \rceil$
7. while $\sum_{k=1}^{M} b_k < R_l$
8. find $k^* \in \{1,..,M\}$ s.t

9. $(1) b_{k^*} < b^{\max}$
10. $(2) \Delta e_{k^*}(b_{k^*}+1) = \min_{q: b_q < b^{\max}} \Delta e_q (b_q + 1)$
11. set $(1) E_{acc} = E_{acc} + \Delta e_{k^*}(b_{k^*}+1)$
12. $(2) b_{k^*} = b_{k^*} + 1$
13. end
14. $E_l = l.(E_{acc} + E_c) - (l.R_l - R).\Delta e_{k^*}(b_{k^*})$
15. $I_l = k^*$
16. $B_{lk} = b_k ; 1 \le k \le M$
17. $l = l - 1$
18. end
19. $l^* = \underset{1 \le l \le L}{\text{argmin}}\ E_l$
20. $E_{l^*} = \min_{1 \le l \le L} E_l$
21. $b_{ik} = B_{l^* k};\ 1 \le k \le M, 1 \le i \le l^*$
22. $b_{ik} = b_{ik} - 1; k = I_{l^*}, 1 \le i \le (l^*.R_{l^*} - R)$

The previous algorithm looks for the best number of OFDM symbols to use in order to minimize the total energy. It has a very small overhead compared to the standard greedy algorithms in [2]. The algorithm works as a normal greedy discrete bit loading algorithm, but remembers the total energy assuming *l* number of symbols are used for data transmission, where $1 \le l \le L$. We use the fact that bit distribution over the *l* OFDM symbols will be almost identical and apply the greedy algorithm on R_l bits (steps 6-13), then the energy used to fill the extra $l.R_l - R_l$ bits is subtracted in step 14. The location of these extra bits is stored in step 15 and the current bit distribution is stored in step 16. After finishing the bit assignments assuming one OFDM symbol, the total energies required to send the data over all possible number of symbols are compared, and the number of OFDM symbols used to transmit with minimum energy number of symbols is selected in steps 19-21. The bit distribution over the l^* OFDM symbols is adjusted in step 22 to remove the extra $l^*. R_l^* - R$ bits that were assigned during the algorithm.

IV. Rate Maximization Problem

In this problem, the goal is to maximize the total number of transmitted bit using a limited energy budget [2]:

$$\begin{aligned} &\max_{0 \le b_k \le b^{\max}} \sum_{k=1}^{M} b_k \\ &s.t \ \sum_{k=1}^{M} e_k(b_k) \le E_{tot};\ b_k \le b^{\max}, k = 1,...,M \end{aligned} , \qquad (9)$$

Similar to the margin maximization problem, if circuit energy is not considered, a greedy algorithm in [2] can be used to solve this problem. The same algorithm can be used if we considered the possibility of sending over multiple OFDM symbols. But when we take into account the circuit energy, the greedy algorithm can not determine the optimal number of OFDM symbols to use. Based on Lemma 1, a circuit energy aware bit loading algorithm is described below:

Algorithm 3:

1. $b_k = 0;\ 1 \le k \le M$
2. $l = L$
3. $R_l = 0;\ 1 \le l \le L$
4. $R_{acc} = 0$
5. while $(l > 0)$
6. $E_l = E_{tot} / l - E_c$
7. while it exists $k \in \{1,...,M\}$ $s.t.$ $b_k < b^{\max}$
8. find $k^* \in \{1,...,M\}$ $s.t.$
9. $\Delta e_{k^*}(b_{k^*}+1) = \min_{q: b_q < b^{\max}} \Delta e_q (b_q + 1)$
10. $(1) R_{acc} = R_{acc} + 1$
11. $(2) b_{k^*} = b_{k^*} + 1$
12. if $\sum_{k=1}^{M} e_k(b_k) \ge E_l$ then goto 15
13. end
14. end
15. $R_l = l.R_{acc} - \left\lceil \left(l \sum_{k=1}^{M} e_k(b_k) - (E_{tot} - l.E_c) \right) / \Delta e_{k^*}(b_{k^*}) \right\rceil$
16. $I_l = k^*$
17. $B_{lk} = b_k;\ 1 \le k \le M$
18. $l = l - 1$
19. end
20. $l^* = \underset{1 \le l \le L}{\arg\max}\ R_l$
21. $R_{l^*} = \max_{1 \le l \le L} R_l$
22. $b_{ik} = B_{l^* k};\ 1 \le k \le M, 1 \le i \le l^*$
23. $b_{ik} = b_{ik} - 1;\ k = I_{l^*},\ 1 \le i \le l^* \sum_{k=1}^{M} B_{kl^*} - R_{l^*}$

The algorithm works in a way similar to Algorithm 2, but instead of trying to find the minimum energy E_{l^*} , it tries to find the maximum possible number of bits to transmit R_{l^*} for a given total energy budget E_{tot} . A greedy algorithm assuming the available energy is only E_l is applied in steps 6-15. Step 16 determines the maximum number of bits that can be transmitted when using *l* OFDM symbols. In steps 21-24, we choose the best number of OFDM symbols to solve the rate maximization problem.

V. Numerical Results

We present some numerical results that show the potential gain when using the circuit energy aware greedy algorithm. We assume an uncoded multicarrier system with system parameters in table I with σ_n^2 = -174 dBm/Hz.

TABLE I: System Parameters

L = 10	G_l = 30dB	BW = 15MHz
B_{max} = 8	κ = 3.5	N_f =3dB (Noise figure)
M = 48	E_c = 300 mW	M_l =1

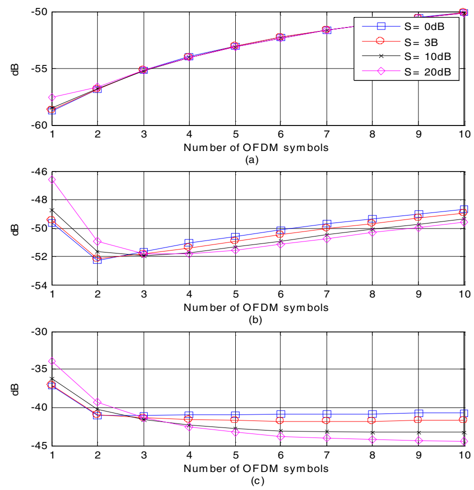

Fig. 4. Average energy per bit for (a) *d*=1m, (b) *d*=3m and (c) *d*=8m.

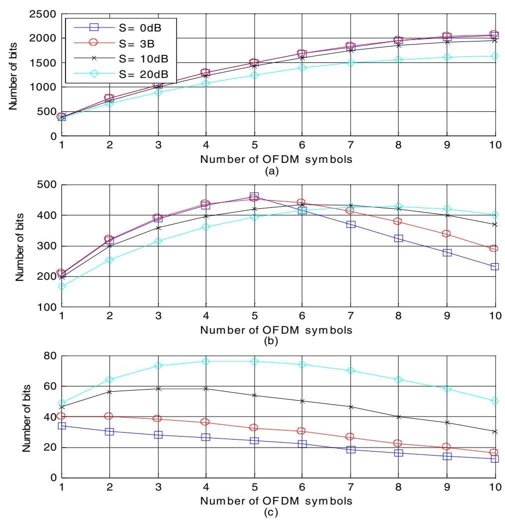

Fig. 5. Average energy per bit for (a) *d*=1m, (b) *d*=3m and (c) *d*=8m.

We start by showing the potential gain in the margin maximization problem. Fig 4 shows the average energy per bit when using different number of OFDM symbols to transmit 192 bits. In case of short distances (*d*=1 m), then regardless of the frequency selectivity of the channel, the circuit energy will dominate the overall energy and sending all the bits using only 1 OFDM symbol will be the most energy efficient technique. As the distance between the Tx and the Rx increases (*d*=3 m), the transmission power component will be in the same range of the circuit energy component. At large distances (*d*=8 m), the circuit energy component will be negligible compared to the transmission energy and using all symbols will always lead to the minimum total energy or very close to it. Note that if we have not considered the circuit energy, the best solution will be always using the 10 possible OFDM symbols. In case of short distance, our algorithm can achieve savings of 9 dB compared to the conventional discrete bit loading algorithms that does not take the circuit energy into account.

In case of rate maximization, the transmitter has a limited energy budget and tries to maximize the total number of bits that are transmitted in a signaling period. Fig. 5 shows the results with a total energy budget of $15E_c$. For (*d*=1 m), the energy used for transmission is enough to send large number of bits in all available subcarriers. As a result, sending over the 10 OFDM symbols will lead to the maximum number of transmitted bits. On the other hand, at (*d*=8 m), the transmitter will tend to send over less number of OFDM symbols to keep more energy for data transmission. Note that for the highly selective channel with *S*=20 dB, the transmitter will try to send over more number of OFDM symbols to get the gain of good channels in all symbols. As a result, the optimal number of symbols is 5, which is larger than the rest of the channels. Note that if the circuit energy was not taken into account, Algorithm 1 would always choose to send over 10 OFDM symbols. At large distances with limited power budget, our algorithm selects the optimal number of OFDM symbols and can achieve gains between 50% in case of *S*=20dB and 180% in case of *S*=0 dB.

VI. Conclusion

In this paper, both margin maximization and rate maximization problems were solved using circuit energy aware bit loading algorithms. We showed that energy savings in order of 9 dB can be achieved in the margin maximization problem. 180% increase in the number of transmitted bits can be achieved in the rate maximization problem. The algorithms have very little overhead compared to circuit energy unaware algorithms.

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